IRREVERSIBLE LOSSES OF PROCESS VARIATION

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Traditional methods for estimating benefits rely on statistical measures of process variation and acceptable deviation from specifications. In this paper is presented a deterministic formula for calculating product loss that results from the variation itself. The formula takes into account the effects of process conditions and post-processing blending. By assuming infinite blending capacity, this formula represents a lower bound on the product losses that actually can occur. The formula derived for irreversible loss is applied to a couple of example problems. The first example is a simple two-component flash. Results from this example show that there is an irreversible loss of process variation even with linear blending properties. The second example shows the losses associated with variation in the cloud point of a diesel product in a multi-cut fractionator. This example illustrates the additional losses that result from nonlinear blending characteristics.

1 INTRODUCTION

Traditional methods of benefit analysis for process control rely on statistical measures of variation and acceptable frequency of deviation. This paper presents a conservative method for calculating the loss of product that is due to the variation itself.

2 DERIVATION

Two basic principles are used to derive a formula for irreversible loss. The first principle is that product quality and yield are related. The second principle is that excursions away from specifications can be corrected by moving the process temporarily to the other side of the excursion.

It is almost universally true that higher quality comes at the cost of lower yield. An active product specification is really a limitation on production. For a separation process it is possible to make more valuable product by adding impurities, while making a product too clean puts valuable components in less-valuable streams, resulting in quality "giveaway". The general formulation for irreversible loss relies on a yield function to express this relationship between yield and purity.

(1) $Yield = f(z), [z = quality]$

During the time of product processing it is not necessary to be on-spec at every instant for the final product to be on-spec. Brief periods of off-spec production can be compensated for by running with quality giveaway for a period of time and vice versa. This blending of products over time is illustrated in Figure 1.

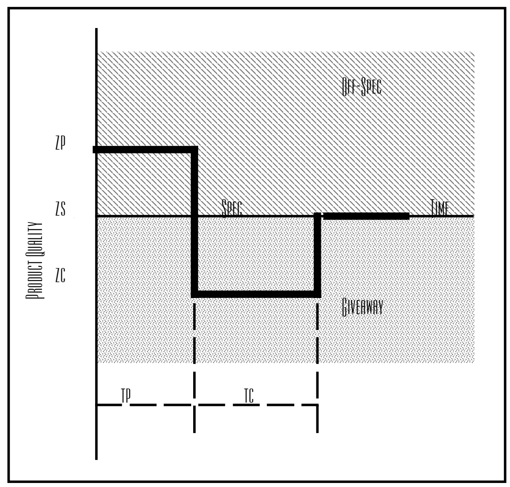

FIGURE 1: Quality Blending Over Time

Not all qualities blend linearly. The common method to account for nonlinear blending is to transform the quality to a blending index which does blend linearly (with respect to mass or volume). In this paper the blending index is represented as a function g(z).

(2) $BlendIndex = g(z), [z = quality]$

The final blended quality is just a yield average of the blending indices. For the production profile in Figure 1, a final product meeting specification is produced by running the process in quality giveaway mode for a period of time tc to compensate for running the product off-spec for a period of time tp.

(3) $g(z_s) = \dfrac{t_p f(z_p) g(z_p) + t_c f(z_c) g(z_c)}{t_p f(z_p) + t_c f(z_c)}$

The final blended quality is the same with a varying or constant quality profile. However, when a product is made off-spec, the yield relationship shows that more product is being made. To compensate, a higher quality product must be produced. Since yield is smaller for higher quality, it takes a longer time to produce an equivalent amount of higher quality product, resulting in a net yield loss. This is expressed as a rate loss over time period tp.

(4)
$$RateLoss = \frac{(t_p + t_c) f(z_s) - t_p f(z_p) - t_c f(z_c)}{t_p}$$

or equivalently:

(5) $$RateLoss = f(z_s) - f(z_p) + \frac{f(z_s) - f(z_c)}{x}$$

where x = tp/tc.

An infinite number of possibilities exist for blending away the quality excursion that occurred over time t_p. A product of much higher quality can be produced for a shorter time or a slightly better quality product can be produced for a longer time. The minimum product loss is realized by using infinite time to correct the quality excursion with product infinitesimally better than the quality spec. This portion of the rate loss is irreversible and can be expressed as,

(6)
$$IRL = \frac{\lim}{x \to 0} \left\{ f(z_s) - f(z_p) + \frac{f(z_s) - f(z_c)}{x} \right\}$$

where IRL is defined as the irreversible rate loss and f(zc) is subject to the equality defined in equation (3) (rewritten in terms of x)

(7) $$g(z_s) = \frac{x f(z_p) g(z_p) + f(z_c) g(z_c)}{x f(z_p) + f(z_c)}$$

This is really the substance of the equation for irreversible rate loss. What follows are just the mathematical details.

The first two terms of equation (6) are constants, while the numerator and denominator of the quotient both approach 0 as x approaches 0. Since both numerator and denominator are differentiable, L'Hospital's rule can be used to find the limit of the quotient.

$$\frac{\lim}{x \to 0} \frac{f(z_s) - f(z_c)}{x} =$$

(8) $$\frac{\frac{d}{dx}\left[f(z_s) - f(z_c)\right]}{\frac{d}{dx} x} = -\frac{d}{dx} f(z_c)$$

This limit can be found by differentiating equation (7) implicitly. First rearranging, we see that the whole equation is just a function of two variables, zc and x (zp and zs are constants).

(9)
$$x f(z_p) g(z_p) + f(z_c) g(z_c) - g(z_s)\left[x f(z_p) + f(z_c)\right] = 0$$

Differentiating implicitly with respect to x,

(10) $$f(z_p) g(z_p) + f(z_c) \frac{dg(z_c)}{dz_c} \frac{dz_c}{dx} + g(z_c) \frac{df(z_c)}{dz_c} \frac{dz_c}{dx} - g(z_s) f(z_p) - g(z_s) \frac{df(z_c)}{dz_c} \frac{dz_c}{dx} = 0$$

and then solving for dzc/dx yields the following:

(11) $$\frac{dz_c}{dx} = \frac{g(z_s) f(z_p) - f(z_p) g(z_p)}{f(z_c) \frac{dg(z_c)}{dz_c} + g(z_c) \frac{df(z_c)}{dz_c} - g(z_s) \frac{df(z_c)}{dz_c}}$$

This result can then be used to obtain a formula for the derivative expressed in equation (8).

(12)
$$-\frac{d}{dx} f(z_c) = -\frac{df(z_c)}{dz_c} \frac{dz_c}{dx} = -\frac{\frac{df(z_c)}{dz_c}\left[g(z_s) f(z_p) - f(z_p) g(z_p)\right]}{f(z_c) \frac{dg(z_c)}{dz_c} + g(z_c) \frac{df(z_c)}{dz_c} - g(z_s) \frac{df(z_c)}{dz_c}}$$

As x approaches zero, zc approaches zs and equation (12) simplifies to the following:

(13)
$$\frac{\lim}{z_c \to z_s} \left[-\frac{d}{dx} f(z_c)\right] = -\frac{\frac{df(z_c)}{dz} f(z_p)\left[g(z_s) - g(z_p)\right]}{f(z_c) \frac{dg(z_s)}{dz}}$$

Substituting this result into equation (6) gives the final form for irreversible loss.

(14)

$$IRL = f(z_s) - f(z_p) - \frac{f'(z_s) f(z_p) [g(z_s) - g(z_p)]}{f(z_s) g'(z_s)}$$

It deserves final mention what this formula actually means. It is the product loss resulting from dynamically making a product away from specification that cannot be recovered by adjusting the process to compensate. Since this formula assumes perfect compensation with infinite blending capacity, it represents a lower bound on the actual product losses that can occur.

3 TWO COMPONENT FLASH

The first example is chosen for its simplicity and reproducibility. A stream of normal butane contains 6 wt% of a propane impurity. It is heated by heat exchange with another stream and then flashed at a constant 240 psia in order to purify the liquid flash stream to 5 wt%. Variations in reboiler medium lead to imprecise control of heat input. A diagram for this system is shown in Figure 2.

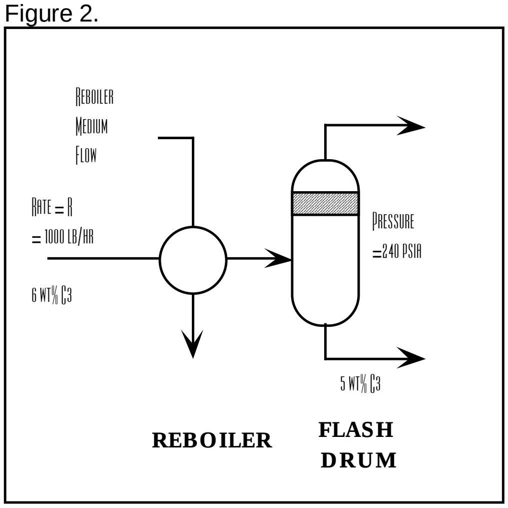

FIGURE 2: C3/C4 Flash

As more heat is added, the concentration of C3 impurity in the liquid product decreases, but so does the flow of liquid product. A series of flash calculations shows how yield and purity are related to varying amounts of heat input (Figure 3).

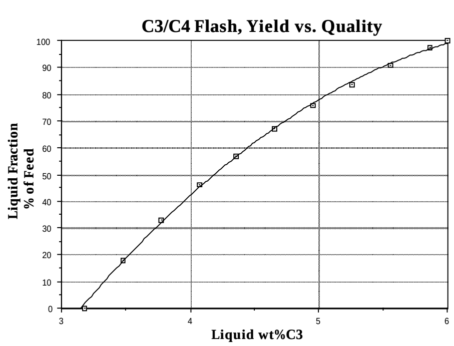

FIGURE 3: Flash Yield vs. C3 Impurity

A second order polynomial curve fit is used to get a yield equation for liquid product as a function of C3 concentration.

(15) $f(z) = -248.66 + 102.25z - 7.3853z^2$

Since a weight composition blends linearly with weight, a linear blending function is used,

(16) $g(z) = z$

and this results in the following graph for irreversible rate loss in the system.

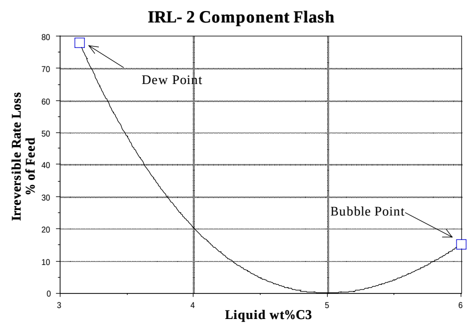

FIGURE 4: IRL for Flash Problem

There are some general properties of this function that can be pointed out. First, loss is always positive. No matter which side of the specification the process deviates on, the net result is a loss of valuable product.

Also. from Figure 3, the yield of liquid at product specification is 78%. When the product is at its dew point or superheated, all of the liquid product is now going overhead into the vapor stream. The irreversible loss at the dew point is also 78%, showing that this loss is completely irreversible.

One way to visualize the total irreversible loss from a specific disturbance is to accumulate the total pounds of loss over time. For a continuous quality measurement this would result in the following equation.

(17) $$CL = \int_{t=0}^{t=T} R \bullet IRL(z) dt$$

where CL is the cumulative product loss and R is the feed rate (see Figure 2). For a sampled data system the cumulative loss is represented by the following equation:

(18) $$CL = \sum_0^T R \bullet IRL(z) \Delta t$$

The cumulative irreversible loss of product from some simulated process data is shown in Figure 5.

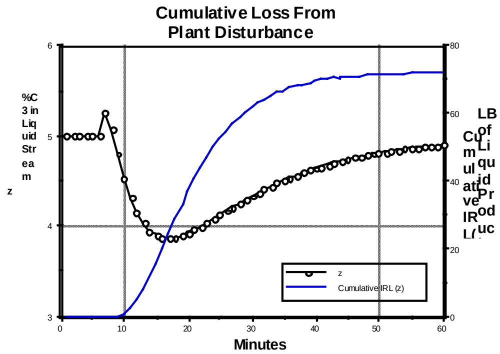

FIGURE 5: Cumulative IRL for C3/C4.

For a flow rate of 1000 lb/hr of feed, this graph shows an irreversible loss of 72 pounds of liquid product over a one-hour period.

4 MULTI-CUT FRACTIONATOR

A second example shows the effect of nonlinear blending on irreversible loss. Multi-cut fractionators are used to separate petroleum fractions with a wide range of boiling points. Increased production of valuable middle distillate streams (kerosine, diesel) are limited by specifications on "cold properties". These specifications on cloud, pour and freeze points are most directly affected by the amount and character of higher boiling point material in the product stream. The more product that is pulled from lower in the column, the higher the cold point.

Specifications on fractionator streams vary widely depending on the economics and the local market being served. Yield models also depend on the feed makeup. For example, feed coming from a hydrocracker will have a much higher middle distillate yield than pure crude oil. For this example, it is assumed that a diesel cut with a 10 Deg F cloud point specification yields 20% of the feed. It is also assumed that increasing diesel draw will increase yield at a rate of 0.4% of feed for every degree of cloudpoint. So the yield model used for this problem is the following:

(19) $$f(z) = 16 + 0.4z$$

where z is cloud point and f is expressed in percent of feed.

Hu and Burns developed a blending index system that predicts cold properties of blends using the following formula:

(20) $$g(z) = 10{,}000 \left\{ \frac{z + 459.69}{140 + 459.69} \right\}^{\frac{1}{\omega}}$$

For cloud point, they concluded the best industry-wide results were obtained using a value of 0.05 for ω. This results in the following equations for g(z) and g'(z):

(21) $$g(z) = 10{,}000 \left\{ \frac{z + 459.69}{140 + 459.69} \right\}^{20}$$

(22) $$g'(z) = 20 \bullet 10{,}000 \left\{ \frac{z + 459.69}{140 + 459.69} \right\}^{19}$$

This is all that is needed to generate the IRL function. This function for zs = 10 is plotted in Figure 6.

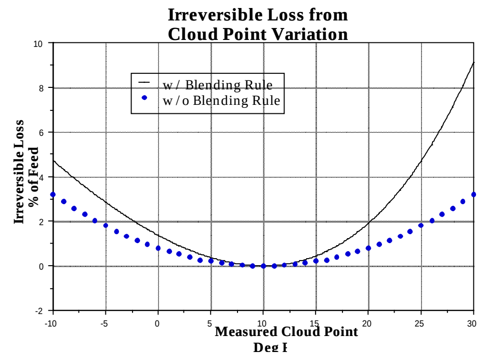

FIGURE 6: IRL with variation in Cloud Point. zs = 10 Deg F.

To compare the effect of nonlinear blending characteristics, the IRL function for linear blending (g(z) = z) is also plotted in this graph. A couple of differences are obvious. First, nonlinear blending increases irreversible loss on the high and low sides. Also, the function with blending is skewed, with the irreversible loss greater for going off-spec on the high side.

5 CONCLUSIONS

In this paper, a formula for irreversible loss is derived and applied to a couple of example problems. Its derivation assumes perfect compensation for any disturbances using infinite blending capacity and is therefore a lower bound on actual product losses.

This formula possesses a number of positive characteristics. Its deterministic nature makes it unnecessary to have a statistically significant amount of data for analysis. It is in fact quite useful for analyzing data that might be discarded by traditional techniques. It also gives some quantifiable justification for running a plant smoother, even when the final product is blended to be on-spec.

6 DEFINITION OF TERMS

CL	Cumulative product loss
f	Yield function
g	Blending index
IRL	Irreversible rate loss
R	Feed flow rate
t	time
tp	time of production
tc	time of compensation
x	Ratio of time, compensation/production
z	Constraining quality measurement
zs	Quality specification
zp	Production quality
zc	Compensation quality

7 LITERATURE CITED

Hu, J., and Burns, A.M., "New Method Predicts Cloud, Pour and Flash Points of Distillate Blends", Hydrocarbon Processing, Vol. 49, No. 11 (November, 1970), pp.213-216.